

MATRIX BINOMIALS

SL TYPE I

Let $X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$. Calculate $X^2, X^3, X^4; Y^2, Y^3, Y^4$

By considering integer powers of X and Y , find expressions for $X^n, Y^n, (X + Y)^n$.

Let $A = aX$ and $B = bY$, where a and b are constants.

Use different values of a and b to calculate $A^2, A^3, A^4; B^2, B^3, B^4$

By considering integer powers of A and B , find expressions for $A^n, B^n, (A + B)^n$.

Now consider $M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$.

Show that $M = A + B$, and that $M^2 = A^2 + B^2$.

Hence, find the general statement that expresses M^n in terms of aX and bY .

Test the validity of your general statement by using different values of a, b , and n .

Discuss the scope and/or limitations of your general statement.

Use an algebraic method to explain how you arrived at your general statement.